

Vector Algebra

Scalar Triple Product

Basic Identity:

$$\begin{aligned}[A, B, C] &\equiv A \circ (B \times C) \\ &= B \circ (C \times A) \\ &= C \circ (A \times B) \\ &= \det(A \ B \ C) \\ &= \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}\end{aligned}$$

Additional Identities:

$$[A, B, C]D = [D, B, C]A + [A, D, C]B + [A, B, D]C$$

Vector Triple Product

Basic Identity:

$$A \times (B \times C) = B(A \circ C) - C(A \circ B)$$

Additional Identities:

$$\begin{aligned}(A \times B) \times C &= -C \times (A \times B) \\ &= -A(B \circ C) + B(A \circ C)\end{aligned}$$

Vector Quadruple Product

Basic Identity (*also known as Lagrange's Identity*):

$$(A \times B) \circ (C \times D) = (A \circ C)(B \circ D) - (A \circ D)(B \circ C)$$

Additional identities :

Using $A^2 = A \circ A$ and $[A, B, C]$ as the scalar triple product $A \circ (B \times C)$

$$\begin{aligned}(A \times B)^2 &= A^2 B^2 - (A \circ B)^2 \\ A \times (B \times (C \times D)) &= B(A \circ (C \times D)) - (A \circ B)(C \times D) \\ (A \times B) \times (C \times D) &= (C \times D) \times (B \times A) \\ &= [A, B, D]C - [A, B, C]D \\ &= [C, D, A]B - [C, D, B]A\end{aligned}$$