Weighted Least Squares Regression

With any of the least squares regression methods, weighting may be applied to each point to up-weight or down-weight the significance of each point. Expanding on the concept of quadratic regression, this section explains how to apply weighting to each point.

Starting with the quadratic equation:

\[ y = Ax^2 + Bx + C \]

The coefficients of the equation \((A, B, \text{and } C)\) can be solved for by:

\[
\begin{bmatrix}
A \\
B \\
C
\end{bmatrix} = \begin{bmatrix}
\sum x^4 & \sum x^3 & \sum x^2 \\
\sum x^3 & \sum x^2 & \sum x \\
\sum x^2 & \sum x & n
\end{bmatrix}^{-1} \begin{bmatrix}
\sum x^2 y' \\
\sum xy' \\
\sum y'
\end{bmatrix}
\]

*Note: Derivation of the quadratic least squares regression can be found [here](#).*

To apply weighting, each point \((x_i, y_i)\) is multiplied by a weight \(w_i\).

\[
\begin{bmatrix}
A \\
B \\
C
\end{bmatrix} = \begin{bmatrix}
\sum x_i^4 w_i & \sum x_i^3 w_i & \sum x_i^2 w_i \\
\sum x_i^3 w_i & \sum x_i^2 w_i & \sum x_i w_i \\
\sum x_i^2 w_i & \sum x_i w_i & \sum w_i
\end{bmatrix}^{-1} \begin{bmatrix}
\sum x_i^2 y' w_i \\
\sum x_i y' w_i \\
\sum y' w_i
\end{bmatrix}
\]

The weight \(w_i\) controls how many times a point is applied to the regression. For instance, consider a quadratic that is defined by 5 points, and points 1-4 all have a weight of 1. The effect that point 5 has on the solution is driven by its weight. This effect is shown below.

C++ pseudocode of point weighting is shown applied to the polynomial least squares regression below.

```cpp
//Declare the calculation arrays
double EqnArray[Order+1];
double CoeffArray[Order+1][Order+1];
double SolnArray[Order+1];

//Calculate the polynomial variables for the current point. For the quadratic
//y = Ax^2 + Bx + C the EqnArray = [x^2 , x , 1]
EqnArray[Order] = 1;
for(int i=Order-1 ; i>=0 ; i--)
   EqnArray[i] = EqnArray[i+1]*x;

for(int n=0 ; n<NumPoints ; n++)
{
   for(int i=0 ; i<=Order ; i++)
   
   
```
{ 
    //Accumulate points in the solution array
    SolnArray[i] += EqnArray[i]*y*Weight[n];

    //Accumulate points in the coefficient array
    for(int j=0; j<Order; j++)
        CoeffArray[i][j] += EqnArray[i]*EqnArray[j]*Weight[n];
}

Property of ahinson.com  –  Last Updated December 9, 2011