**Weighted Least Squares Regression**

With any of the least squares regression methods, weighting may be applied to each point to up-weight or down-weight the significance of each point. Expanding on the concept of quadratic regression, this section explains how to apply weighting to each point.

Starting with the quadratic equation:

$$y = Ax^2 + Bx + C$$

The coefficients of the equation ($A$, $B$, and $C$) can be solved for by:

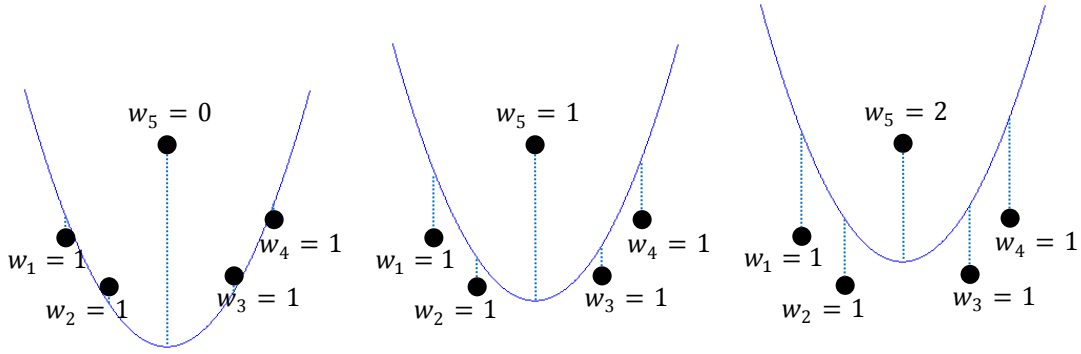
$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} \sum x^4 & \sum x^3 & \sum x^2 \\ \sum x^3 & \sum x^2 & \sum x \\ \sum x^2 & \sum x & n \end{bmatrix}^{-1} \cdot \begin{bmatrix} \sum x^2 y' \\ \sum x y' \\ \sum y' \end{bmatrix}$$

*Note: Derivation of the quadratic least squares regression can be found here.*

To apply weighting, each point $(x_i, y_i)$ is multiplied by a weight $w_i$.

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} \sum x_i^4 w_i & \sum x_i^3 w_i & \sum x_i^2 w_i \\ \sum x_i^3 w_i & \sum x_i^2 w_i & \sum x_i w_i \\ \sum x_i^2 w_i & \sum x_i w_i & \sum w_i \end{bmatrix}^{-1} \cdot \begin{bmatrix} \sum x_i^2 y' w_i \\ \sum x_i y' w_i \\ \sum y' w_i \end{bmatrix}$$

The weight $w_i$ controls how many times a point is applied to the regression. For instance, consider a quadratic that is defined by 5 points, and points 1-4 all have a weight of 1. The effect that point 5 has on the solution is driven by its weight. This effect is shown below.




C++ pseudocode of point weighting is shown applied to the polynomial least squares regression below.

```
//Declare the calculation arrays
double EqnArray[Order+1];
double CoeffArray[Order+1][Order+1];
double SolnArray[Order+1];

//Calculate the polynomial variables for the current point. For the quadratic
//y = Ax^2 + Bx + C the EqnArray = [x^2 , x , 1]
EqnArray[Order] = 1;
for(int i=Order-1 ; i>=0 ; i--)
  EqnArray[i] = EqnArray[i+1]*x;

for(int n=0 ; i<NumPoints ; i++)
{
  for(int i=0 ; i<=Order ; i++)
```

```
  {
    //Accumulate points in the solution array
    SolnArray[i] += EqnArray[i]*y*Weight[n];

    //Accumulate points in the coeffient array
    for(int j=0 ; j<=Order ; j++)
      CoeffArray[i][j] += EqnArray[i]*EqnArray[j]*Weight[n];

  }
}
```