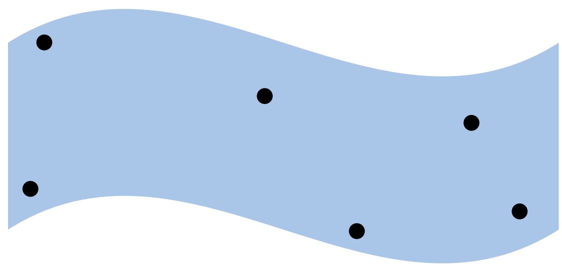
**Polynomial Surface Fit**

Polynomial surface fit determines the equation of an $m^{th}$ x $n^{th}$ order polynomial surface that passes through n*m+1 points. Unlike bi-polynomial regression, this method requires n*m+1 data points and the resulting polynomial surface will pass through every point.



The image above represents a $2^{nd}$ order surface with the equation:

$$z = Ax^2 + Bxy + Cy^2 + Dx + Ey + F$$

To define this surface there must be 6 linearly independent points. To solve for the coefficients, the same process from the curve fitting is used. The surface is represented as 6 linear equations and solved simultaneously.

$$\begin{aligned}
Ax_1^2 + Bx_1y_1 + Cy_1^2 + Dx_1 + Ey_1 + F = z_1 \\
Ax_2^2 + Bx_2y_2 + Cy_2^2 + Dx_2 + Ey_2 + F = z_2 \\
Ax_3^2 + Bx_3y_3 + Cy_3^2 + Dx_3 + Ey_3 + F = z_3 \\
Ax_4^2 + Bx_4y_4 + Cy_4^2 + Dx_4 + Ey_4 + F = z_4 \\
Ax_5^2 + Bx_5y_5 + Cy_5^2 + Dx_5 + Ey_5 + F = z_5 \\
Ax_6^2 + Bx_6y_6 + Cy_6^2 + Dx_6 + Ey_6 + F = z_6
\end{aligned}$$

In matrix form:

$$\begin{bmatrix} x_1^2 & x_1y_1 & y_1^2 & x_1 & y_1 & 1 \\ x_2^2 & x_2y_2 & y_2^2 & x_2 & y_2 & 1 \\ x_3^2 & x_3y_3 & y_3^2 & x_3 & y_3 & 1 \\ x_4^2 & x_4y_4 & y_4^2 & x_4 & y_4 & 1 \\ x_5^2 & x_5y_5 & y_5^2 & x_5 & y_5 & 1 \\ x_6^2 & x_6y_6 & y_6^2 & x_6 & y_6 & 1 \end{bmatrix} \cdot \begin{bmatrix} A \\ B \\ C \\ D \\ E \\ F \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \end{bmatrix}$$

Solve for A-F by taking the inverse of the coefficient matrix.

$$\begin{bmatrix} A \\ B \\ C \\ D \\ E \\ F \end{bmatrix} = \begin{bmatrix} x_1^2 & x_1y_1 & y_1^2 & x_1 & y_1 & 1 \\ x_2^2 & x_2y_2 & y_2^2 & x_2 & y_2 & 1 \\ x_3^2 & x_3y_3 & y_3^2 & x_3 & y_3 & 1 \\ x_4^2 & x_4y_4 & y_4^2 & x_4 & y_4 & 1 \\ x_5^2 & x_5y_5 & y_5^2 & x_5 & y_5 & 1 \\ x_6^2 & x_6y_6 & y_6^2 & x_6 & y_6 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \end{bmatrix}$$