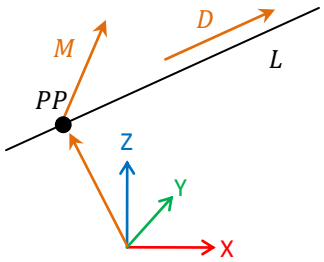
Plücker Line – Principal Point

The principal point (PP) of the Plücker line is the point at which the line passes closest to the origin.



This value can be calculated by taking the cross product of the unit direction vector D and the moment vector M.

$$PP = \widehat{D} \times M$$

To unitize the direction vector the whole line must be divided by the magnitude of the direction vector m.

$$m = \sqrt{D_x^2 + D_y^2 + D_z^2}$$

The line's components are then updated by dividing all the components of the line by m.

$$\widehat{D}_x = \frac{D_x}{m} \qquad M_x = \frac{M_x}{m}$$
$$\widehat{D}_x = \frac{D_y}{m} \qquad M_y = \frac{M_y}{m}$$
$$\widehat{D}_x = \frac{D_z}{m} \qquad M_z = \frac{M_z}{m}$$

This is the standard representation of the Plücker line. The direction vector is stored as a unit vector, and the magnitude of the moment vector is equal to the magnitude of the vector from the origin to the principal point of the line. If the line is stored in standard representation, the unitization step is not required.

To calculate the principal point, take the cross product of the unit direction vector $\widehat{D}$ and moment vector M.

$$PP_x = \widehat{D}_y \cdot M_z - \widehat{D}_z \cdot M_y$$
$$PP_y = \widehat{D}_z \cdot M_x - \widehat{D}_x \cdot M_z$$
$$PP_z = \widehat{D}_x \cdot M_y - \widehat{D}_y \cdot M_x$$