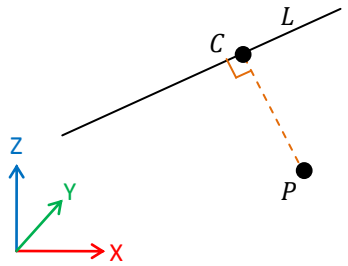
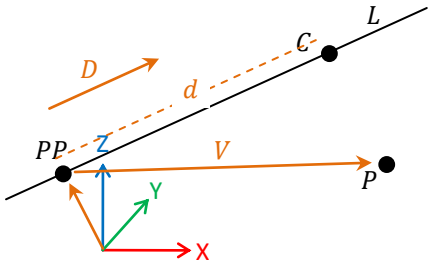
Plücker Line – Point of Closest Approach

To find the point of closest approach to the line (C) for a point in space (P) as shown below:



The principal point (PP) can be used to define a starting point on the line. The distance d along the line from the point PP to C must be calculated.



The distance d could be calculated by projecting the vector V onto the direction vector of the line, but since the origin is normal to the principal point in the plane of $\overline{PP - C - P}$ the value d can be calculated using the vector from the origin to the point.

$$d = V \circ \hat{D} = P \circ \hat{D}$$

The principal point can be calculated as:

$$PP_x = \hat{D}_y \cdot M_z - \hat{D}_z \cdot M_y$$
$$PP_y = \hat{D}_z \cdot M_x - \hat{D}_x \cdot M_z$$
$$PP_z = \hat{D}_x \cdot M_y - \hat{D}_y \cdot M_x$$

The vector V can then be projected onto the line by taking the dot product of the vector and the unit direction vector of the line $\hat{D}$.

$$d = P_x \cdot \hat{D}_x + P_y \cdot \hat{D}_y + P_z \cdot \hat{D}_z$$

The closest point can then be calculated by adding the scaled direction vector to the principal point

$$C_x = PP_x + \hat{D}_x \cdot d$$
$$C_y = PP_y + \hat{D}_y \cdot d$$
$$C_z = PP_z + \hat{D}_z \cdot d$$