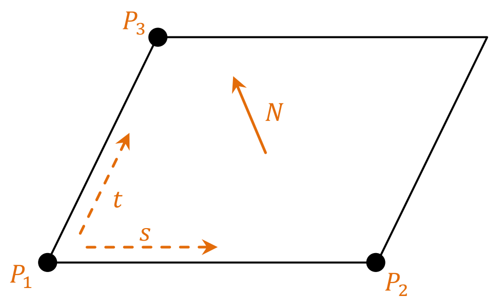
Parametric Plane

A plane can be described as a set of parametric equations from three points P_1, P_2, and P_3 and two parameters s and t.



$$x = x_1 + (x_2 - x_1) \cdot s + (x_3 - x_1) \cdot t$$
$$y = y_1 + (y_2 - y_1) \cdot s + (y_3 - y_1) \cdot t$$
$$z = z_1 + (z_2 - z_1) \cdot s + (z_3 - z_1) \cdot t$$

The normal vector N can be calculated by taking the cross product of the vectors between the points.

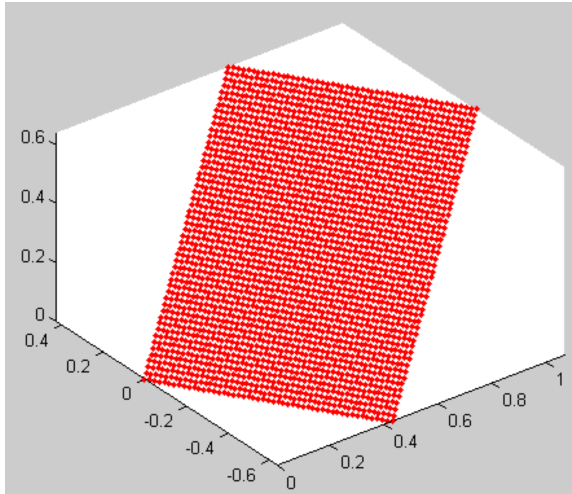
$$N = (P_2 - P_1) \times (P_3 - P_1)$$

When expanded out becomes:

$$N_x = (\mathrm{y}_2 - \mathrm{y}_1) * (\mathrm{z}_3 - \mathrm{z}_1) - (\mathrm{z}_2 - \mathrm{z}_1) * (\mathrm{y}_3 - \mathrm{y}_1)$$
$$N_y = (\mathrm{z}_2 - \mathrm{z}_1) * (\mathrm{x}_3 - \mathrm{x}_1) - (\mathrm{x}_2 - \mathrm{x}_1) * (\mathrm{z}_3 - \mathrm{z}_1)$$
$$N_z = (\mathrm{x}_2 - \mathrm{x}_1) * (\mathrm{y}_3 - \mathrm{y}_1) - (\mathrm{y}_2 - \mathrm{y}_1) * (\mathrm{x}_3 - \mathrm{x}_1)$$

The directions of s and t do not need to be orthogonal, however this is typically suggested for defining a plane. When s and t are parallel, the plane collapses to a line in the direction of s. If s and t are non-parallel, but not orthogonal, the plane will still be uniquely defined, but the point spacing in s and t becomes sheared and less intuitive.

An example parametric plane is shown below, with an orthogonal s and t. The points are sampled evenly in s and t.


0.6
0.4
0.2
0
0.4
0.2
0
-0.2
-0.4
-0.6
0
0.2
0.4
0.6
0.8
1