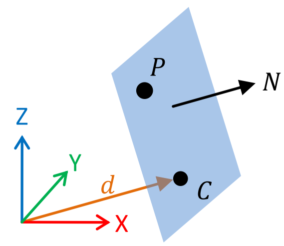
Homogeneous Plane Equations

Homogeneous planes are represented by their normal vector and the distance of closest approach to the coordinate system origin.



Given a normal vector (N) and a point on the plane (P), the full planar equation can be solved with the standard Cartesian formula:

$$Ax + By + Cz + D = 0$$

The direction cosines of the normal vector correspond to the values of A, B, and C and x, y, and z correspond to the value of point P.

$$D = -N_x P_x - N_y P_y - N_z P_z$$

The full planar equation can now be represented in short hand form much like the equation of a point.

$$[D\,;N] \quad or \quad [D\,;A\,,B\,,C]$$

Like the equations of points and lines, the components of a plane can be multiplied by a scalar without affecting the plane.

$$[D\,;A\,,B\,,C] = \alpha \cdot [D\,;A\,,B\,,C] = [\alpha D\,;\alpha A\,,\alpha B\,,\alpha C]$$

A convenient way to represent a homogeneous plane is to unitize the normal vector which causes D to reduce to the negative of the distance from the origin to the point of closest approach.

$$[D\,;A\,,B\,,C] = \left[-d\,;N_x, N_y, N_z\right]$$

Plane from a Point and a Normal Vector

Description: A homogeneous plane can be constructed from the normal vector and any point on the plane.

Given a normal vector V and a point on the plane P the equation of the homogeneous plane can be solved using the planar equation.

$$Ax + By + Cz + D = 0$$

We know that the normal vector is described by the components A, B, and C in the homogeneous equation of a plane. The only value that remains to be solved for is D. Plugging in the normal vectors directions into the planar equation and solving for D gives the new equation:

$$D = -\left(V_x x + V_y y + V_z z\right)$$

The variables x, y, and z represent the location of any point on the plane, so plugging in the point P, the value of D can be solved for.

$$D = -(V_x P_x + V_y P_y + V_z P_z)$$

To put the plane in standard notation, the full planar equation must be divided by the magnitude (m) of the input vector:

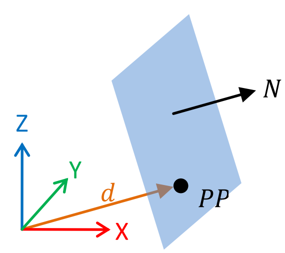
$$m = \sqrt{V_x^2 + V_y^2 + V_z^2}$$

Therefore the standard planar equations are calculated as:

$$A = {V_x}/{m}$$
$$B = {V_y}/{m}$$
$$C = {V_z}/{m}$$
$$D = \frac{-(V_x P_x + V_y P_y + V_z P_z)}{m}$$

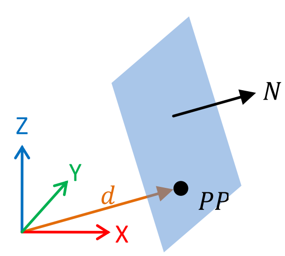
Distance from Origin to Plane

The principal point (PP) of a plane is the point at which the plane is closest to the coordinate system origin. The principal p



Principal Point of a Plane

The principal point (PP) of a plane is the point at which the plane is closest to the coordinate system origin. The principal point can be solved for by extending the normal vector of the plane to the distance from the origin.



The distance d from the origin to the principal point is defined by the D component of the line.

$$d = -D$$

This value is a directional offset distance from the plane. If the value of d is positive, the point is offset along the normal vector from the plane surface. If d is negative the point is in the negative normal direction.