SECTION 6 PHOTOGRAMMETRY

6.1 Geodesy

6.1.1 Nomenclature

[TBD]

6.1.2 Geodetic Coordinate Systems and Datums

[TBD]

6.1.2.1 World Geodetic Survey 1984 (WGS-84)

The World Geodetic System defines a reference ellipsoid for the Earth that is commonly used in geodesy. The latest revision, 1984 (which was last revised in 2004), should be valid up to about 2010.

Geodetic coordinates are defined in latitude, longitude, and elevation with respect to the ellipsoid. The coordinate system origin is at the intersection of the equator and the prime meridian. Latitude measures positive to the north with a max of 90° at the north pole, and negative to the south with a minimum of -90° at the south pole. Longitude measures positive to the east with a maximum of 180°, and negative to the west with a minimum of -180°. Elevation is measured from the surface of the ellipsoid in the outward normal direction. Elevations are measured positive rising outward off of the ellipsoid surface and negative for locations underneath the surface.

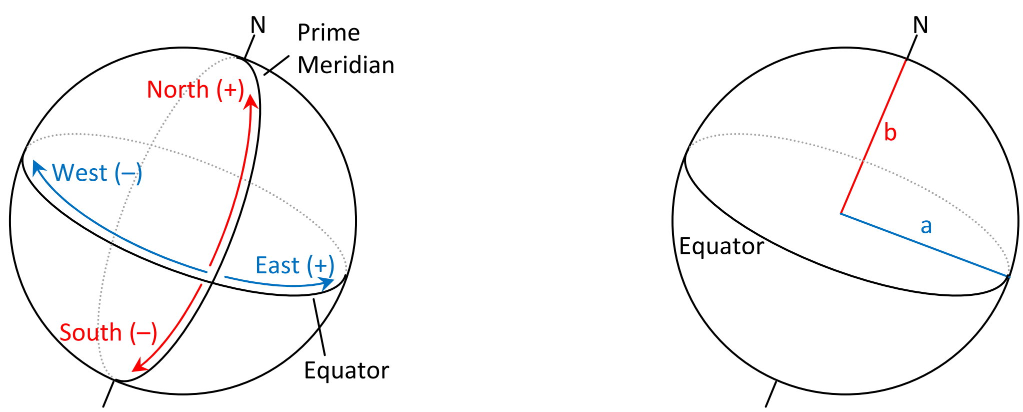


Figure 1: WGS-84 Parameters

The rotation of the Earth causes a slight bulge in the equatorial region with respect to the polar regions. The geometric model that best represents this is an oblate spheroid (basically, a squashed sphere). An oblate spheroid can be described using two quantities: the semi-major axis (a) and the semi-minor axis (b).

For WGS-84, the values of the radii are:

$$a = 6378137.0$$
$$b = 6356752.3142$$

All the remaining ellipsoidal parameters are derived from these two values. The process of converting to and from geodetic coordinates is described in the next several sections.

6.1.2.2 Universal Transverse Mercator (UTM)

[TBD]

6.1.3 Geocentric Coordinate Systems

Geocentric coordinate systems are global Cartesian coordinate frames that have their origins at the center of the Earth. The two common geocentric coordinate systems are detailed below.

6.1.3.1 Earth-Centered, Earth-Fixed

Earth-Centered, Earth-Fixed (ECEF) is a geocentric Cartesian coordinate system with its origin at the center of the Earth. ECEF coordinates have their x-axis piercing the intersection of the equator and the prime meridian, the y-axis crossing the equator at 90° east longitude, and the z-axis pointing at the rotational center of the north pole.

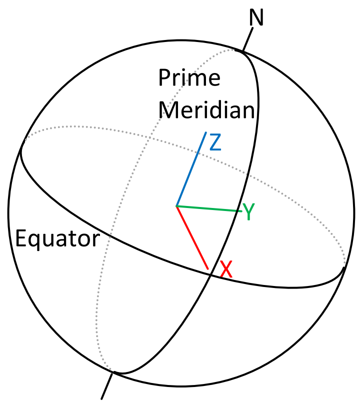


Figure 2: Earth-Centered, Earth-Fixed Coordinate System

The ECEF coordinate system rotates with the Earth and is aligned consistently with respect to the Earth's surface. The ECEF coordinate system is, therefore, useful when dealing with terrestrial objects.

6.1.3.2 Earth-Centered, Inertial

The Earth-Centered, Inertial (ECI) coordinate system is a geocentric coordinate system with its origin at the center of the Earth. The ECI z-axis points through the rotational center of the north pole and it's x-axis points along the vernal equinox. The y-axis is chosen to be orthogonal to the x and z. The vernal equinox is the intersection line between the equatorial plane and the ecliptic (the plane that the planets travel in). This is equivalent to a line from the center of the earth towards the center of the sun on the first day of Spring.

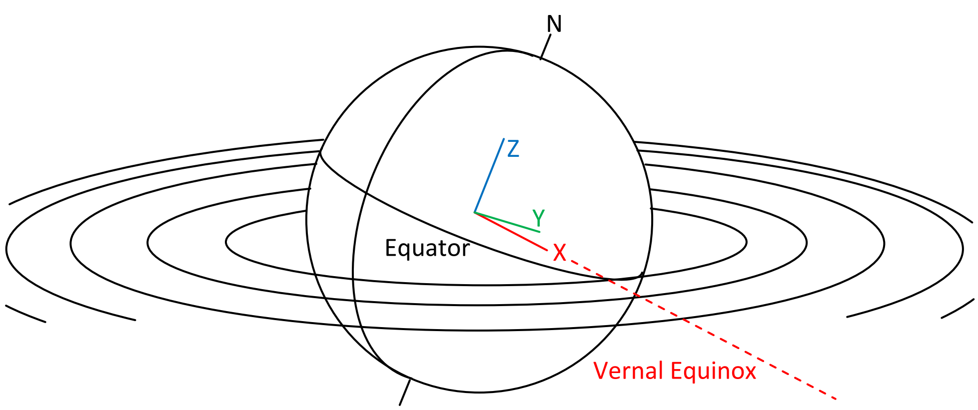


Figure 3: Earth-Centered, Inertial Coordinate System

6.1.4 Topographic Coordinate Systems

Topographic coordinates are Cartesian coordinate systems that are used to reference the space around a point of interest on the Earth. The two typical systems used are East-North-Up and North-East-Down, both are detailed below.

6.1.4.1 East-North-Up (ENU)

The ENU coordinate system is a local Cartesian coordinate system defined for a point on or near the surface of the Earth. The name of the coordinate system indicates the directions of its axes, so for ENU the x, y, and z axes correspond to local east, local north and local up, respectively.

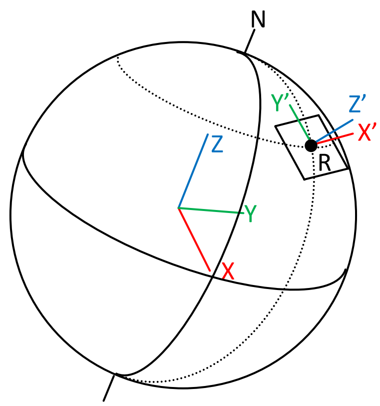


Figure 4: ENU Coordinate System

The ENU system contains a reference point (R) that specifies the coordinate system origin. The ground plane for an ENU system is oriented tangent to the surface of the ellipsoid at the reference point. Conversions to and from ENU are detailed in the next few sections.

6.1.4.2 North-East-Down (NED)

NED is second form of topographic coordinates very similar to ENU. NED is simply a rotated version of ENU where the vertical axis points downward into the Earth and the x and y-axes point to the north and east, respectively.

All topographic calculations in this document use ENU coordinates, but conversion between the two systems can be easily done. An ENU point, $P_{ENU} \equiv (x_{ENU}, y_{ENU}, z_{ENU})$, can be converted to a NED point, $P_{NED} \equiv (x_{NED}, y_{NED}, z_{NED})$, by:

$$\begin{bmatrix} x_{NED} \\ y_{NED} \\ z_{NED} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_{ENU} \\ y_{ENU} \\ z_{ENU} \end{bmatrix}$$

The same transformation matrix works for converting NED to ENU as well.

6.1.5 Geodetic-to-Geocentric Transform

Description: Converts a geodetic coordinate in decimal degrees (WGS-84) to a geocentric coordinate in ECEF.

An input geodetic point $(Lat, Lon, Elev)$ can be converted to a geocentric point (x, y, z) using the WGS-84 radii:

$$a = 6378137.0$$
$$b = 6356752.3142$$

Where a is the semi-minor axis and b is the semi-major axis of the Earth. From these components, the flattening (f) of the Earth can be calculated.

$$f = \frac{a-b}{b}$$

The square of the eccentricity (e^2) is calculated from the flattening.

$$e^2 = 2f - f^2$$

The radius of curvature of the prime vertical (N) is calculated at latitude (Lat).

$$N = \frac{a}{\sqrt{1 - e^2 \sin^2(Lat)}}$$

Finally, the geocentric coordinates can be calculated

$$x = (N + Elev) \cdot \cos(Lat) \cdot \cos(Lon)$$
$$y = (N + Elev) \cdot \cos(Lat) \cdot \sin(Lon)$$
$$z = [N \cdot (1 - e^2) + Elev] \cdot \sin(Lat)$$

The ECEF coordinate system is measured from the center of the Earth with the x-axis pointing through the intersection of the equator and the prime meridian. The z-axis points through the north pole at the center of rotation. The y-axis is orthogonal to x and z, pointing through the equator at 90° east longitude.

6.1.6 Geocentric-to-Geodetic Transform

Description: Converts a geocentric coordinate in ECEF to a geodetic coordinate in WGS-84.

An input geocentric point (x, y, z) can be converted to a geodetic point $(Lat, Lon, Elev)$ using the WGS-84 radii:

$$a = 6378137.0$$
$$b = 6356752.3142$$

Where a is the semi-minor axis and b is the semi-major axis of the Earth. From these components, the flattening (f) of the Earth can be calculated.

$$f = \frac{a-b}{b}$$

The square of the eccentricity (e^2) is calculated from the flattening.

$$e^2 = 2f - f^2$$

The radius of the geocentric point at the equator (p) is calculated by

$$p = \sqrt{x^2 + y^2}$$

A couple of intermediate calculations are made

$$\theta = \operatorname{atan}\left(\frac{z \cdot a}{p \cdot b}\right) \qquad e^{\circ 2} = \frac{a^2 - b^2}{b^2}$$

Finally the geodetic location can be solved for:

$$Lat = \text{atan}\left(\frac{Z + e^{\circ 2} \cdot b \cdot \sin^3(\theta)}{p - e^2 \cdot a \cdot \cos^3(\theta)}\right)$$
$$Lon = \text{atan2}(y, x)$$
$$Elev = \frac{p}{\cos\,(Lat)} - N$$

Where N is the radius of curvature of the prime vertical.

$$N = \frac{a}{\sqrt{1 - e^2 \sin^2(Lat)}}$$

6.1.7 Geocentric-to-Topographic Transform

Description: Converts a point from geocentric coordinates (ECEF) to topographic coordinates (ENU). The topographic coordinate system is defined at a reference point on the ellipsoid with its z-axis pointing directly upward. The y-axis points toward the north pole tangential to the ellipsoid at the reference point. The x-axis points eastward tangential to the ellipsoid at the reference point.

A geocentric point $P_C \equiv (x_C, y_C, z_C)$ can be remapped to a topographic coordinate $P_T \equiv (x_T, y_T, z_T)$ using Cartesian coordinate system transformations.

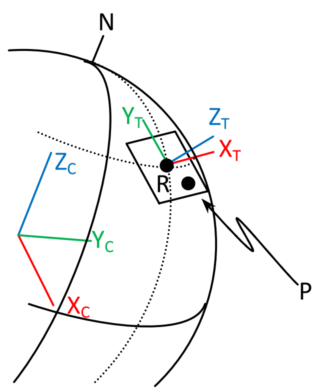


Figure 5: Topographic Coordinate System

The ENU topographic coordinate system is oriented on the ellipsoid such that the z-axis is ascending vertically from the surface of the Earth. The most straight forward way to calculate the $\hat{Z}_T$ vector is to take a slice through the Earth at the given point. This operation returns an ellipse with its major axis aligning with the equator and its minor axis aligning with the poles. This can be done because the ellipsoid is an oblate spheroid and has a consistent horizontal radius all the way around the equator. The elliptical slice is shown below in green.

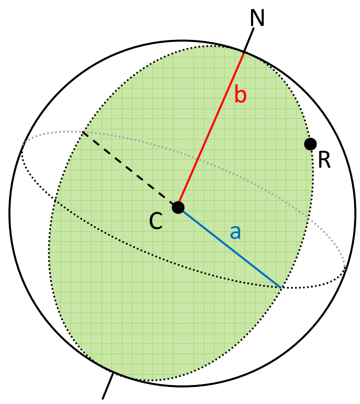


Figure 6: Major and Minor Axes of Local Ellipitical Slice

The normal vector for the edge of a 2D ellipse can be found by taking the bisector angle between the vectors from the foci to the edge of the ellipse.

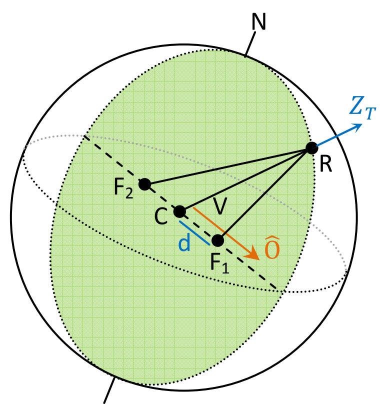


Figure 7: Local Ellipitical Slice Surface Normal

The distance (d) from the center to each focus can be found using the formula:

$$d = \sqrt{a^2 - b^2}$$

The offset from the center to the focus must be applied in the plane of the ellipse. The ellipse passes through the reference point (R), so a projection of the vector to R_C (in geocentric coordinates) onto the equatorial plane will give the direction to one of the foci. The unit offset vector ($\hat{O}$) can be found by unitizing the horizontal components of the vector to point R_C.

$$\hat{O} = \left(\frac{R_{C,x}}{\sqrt{R_{C,x}^2 + R_{C,y}^2}}, \frac{R_{C,y}}{\sqrt{R_{C,x}^2 + R_{C,y}^2}}, 0 \right)$$

The location of each focus can then be calculated by applying the distance (d) to the offset vector in both directions.

$$F_1 = C + d\hat{O}$$
$$F_2 = C - d\hat{O}$$

The unit vectors from the foci to the reference point are then calculated.

$$\hat{V}_1 = \frac{R - F_1}{|R - F_1|}$$
$$\hat{V}_2 = \frac{R - F_1}{|R - F_2|}$$

The bisector of these two angles is taken to get the direction of the topographic unit vector $\hat{Z}_T$.

$$\hat{Z}_T = \frac{(\hat{V}_1) + (\hat{V}_2)}{|(\hat{V}_1) + (\hat{V}_2)|}$$

The x-axis is then calculated by taking the cross product of the $\hat{Z}_T$ vector and the Earth's north vector (which is simply the z-axis of the geocentric coordinate system.

$$\hat{X}_T = \hat{Z}_C \times \hat{Z}_T$$

Where

$$\hat{Z}_C = (0,0,1)$$

Finally the y-axis is calculated to be orthogonal to the x and z-axes.

$$\hat{Y}_T = \hat{Z}_T \times \hat{X}_T$$

The topographic coordinate frame is now defined with the x-axis pointing east, the y-axis pointing north, and the z-axis pointing up from the viewpoint of the reference point.

The conversion from geocentric to topographic is a simple Cartesian coordinate system transfer. A coordinate can be converted from one system to another by a series of simple matrix manipulations. First the point is translated from the geocentric coordinate frame to the topographic (which is centered at point R_C).

$$Trans = \begin{bmatrix} 1 & 0 & 0 & -R_{C,x} \\ 0 & 1 & 0 & -R_{C,y} \\ 0 & 0 & 1 & -R_{C,z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This translation may seem counterintuitive because the point appears to be translating towards the center of the Earth. This is because converting a point to a new coordinate system works exactly opposite of moving a point in the same coordinate system. The movement of a point 10 units along the positive x-axis of a coordinate system appears the same as the coordinate system moving 10 units along the negative x-axis.

The coordinate system must then be rotated into the perspective of the topographic coordinate system. The rotation matrix can be built using the topographic coordinate system's axis vectors as the rows.

$$Rot = \begin{bmatrix} X_{T,x} & X_{T,y} & X_{T,z} & 0 \\ Y_{T,x} & Y_{T,y} & Y_{T,z} & 0 \\ Z_{T,x} & Z_{T,y} & Z_{T,z} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The translation and rotation matrices are combined to form the full coordinate system transform.

$$Xform = \begin{bmatrix} X_{T,x} & X_{T,y} & X_{T,z} & 0 \\ Y_{T,x} & Y_{T,y} & Y_{T,z} & 0 \\ Z_{T,x} & Z_{T,y} & Z_{T,z} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & -R_{C,x} \\ 0 & 1 & 0 & -R_{C,y} \\ 0 & 0 & 1 & -R_{C,z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} X_{T,x} & X_{T,y} & X_{T,z} & (-X_{T,x} \cdot R_{C,x} - X_{T,y} \cdot R_{C,y} - X_{T,z} \cdot R_{C,z}) \\ Y_{T,x} & Y_{T,y} & Y_{T,z} & (-Y_{T,x} \cdot R_{C,x} - Y_{T,y} \cdot R_{C,y} - Y_{T,z} \cdot R_{C,z}) \\ Z_{T,x} & Z_{T,y} & Z_{T,z} & (-Z_{T,x} \cdot R_{C,x} - Z_{T,y} \cdot R_{C,y} - Z_{T,z} \cdot R_{C,z}) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The transform is then simply the matrix product of the geocentric point and the transformation matrix.

$$\begin{bmatrix} x_T \\ y_T \\ z_T \\ 1 \end{bmatrix} = \begin{bmatrix} X_{T,x} & X_{T,y} & X_{T,z} & (-X_{T,x} \cdot R_{C,x} - X_{T,y} \cdot R_{C,y} - X_{T,z} \cdot R_{C,z}) \\ Y_{T,x} & Y_{T,y} & Y_{T,z} & (-Y_{T,x} \cdot R_{C,x} - Y_{T,y} \cdot R_{C,y} - Y_{T,z} \cdot R_{C,z}) \\ Z_{T,x} & Z_{T,y} & Z_{T,z} & (-Z_{T,x} \cdot R_{C,x} - Z_{T,y} \cdot R_{C,y} - Z_{T,z} \cdot R_{C,z}) \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_C \\ y_C \\ z_C \\ 1 \end{bmatrix}$$

6.1.8 Topographic-to-Geocentric Transform

Description: Converts a topographic point in ENU to a geocentric point in ECEF.

Following the logic from the previous section, if topographic point P_T is defined in ENU and a reference point R is defined in ECEF, the geocentric point P_C can be found by essentially undoing the steps cited in the geocentric-to-topographic transform.

First the coordinate system needs to be rotated back to the geocentric frame. This is done by determining the axes of the geocentric coordinate system with respect to the topographic. This simplest way to do this is to take the inverse of the geocentric-to-topographic rotation matrix.

$$Rot^{-1} = \begin{bmatrix} X_{C,x} & X_{C,y} & X_{C,z} & 0 \\ Y_{C,x} & Y_{C,y} & Y_{C,z} & 0 \\ Z_{C,x} & Z_{C,y} & Z_{C,z} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} X_{T,x} & X_{T,y} & X_{T,z} & 0 \\ Y_{T,x} & Y_{T,y} & Y_{T,z} & 0 \\ Z_{T,x} & Z_{T,y} & Z_{T,z} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1}$$

Since the rotation matrix is orthogonal, the inverse can be performed by taking the transpose.

$$Rot^{-1} = \begin{bmatrix} X_{T,x} & X_{T,y} & X_{T,z} & 0 \\ Y_{T,x} & Y_{T,y} & Y_{T,z} & 0 \\ Z_{T,x} & Z_{T,y} & Z_{T,z} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} X_{T,x} & Y_{T,x} & Z_{T,x} & 0 \\ X_{T,y} & Y_{T,y} & Z_{T,y} & 0 \\ X_{T,z} & Y_{T,z} & Z_{T,z} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The translation must then be removed.

$$Trans^{-1} = \begin{bmatrix} 1 & 0 & 0 & R_{C,x} \\ 0 & 1 & 0 & R_{C,y} \\ 0 & 0 & 1 & R_{C,z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

These transformations are made in reverse order so the full transformation matrix is defined as:

$$Xform^{-1} = \begin{bmatrix} 1 & 0 & 0 & R_{C,x} \\ 0 & 1 & 0 & R_{C,y} \\ 0 & 0 & 1 & R_{C,z} \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X_{T,x} & Y_{T,x} & Z_{T,x} & 0 \\ X_{T,y} & Y_{T,y} & Z_{T,y} & 0 \\ X_{T,z} & Y_{T,z} & Z_{T,z} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Xform^{-1} = \begin{bmatrix} X_{T,x} & Y_{T,x} & Z_{T,x} & R_{C,x} \\ X_{T,y} & Y_{T,y} & Z_{T,y} & R_{C,y} \\ X_{T,z} & Y_{T,z} & Z_{T,z} & R_{C,z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This same inverse transformation could have also been obtained by taking the inverse of the original transformation matrix, but this method would have been algebraically cumbersome and would not have shown the relationship between the two transforms.

The geocentric point can be obtained from the topographic point by:

$$\begin{bmatrix} x_C \\ y_C \\ z_C \\ 1 \end{bmatrix} = \begin{bmatrix} X_{T,x} & Y_{T,x} & Z_{T,x} & R_{C,x} \\ X_{T,y} & Y_{T,y} & Z_{T,y} & R_{C,y} \\ X_{T,z} & Y_{T,z} & Z_{T,z} & R_{C,z} \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_T \\ y_T \\ z_T \\ 1 \end{bmatrix}$$

6.1.9 Geodetic-to-Topographic Transform

Description: Converts a geodetic point in WGS-84 coordinates to a topographic point in ENU. Location of ENU coordinate system is defined by a reference point in WGS-84.

The method of converting a geodetic point to a topographic directly is similar to the two stage process of converting to a geocentric point and then to a topographic. The only difference in this method is the knowledge of the reference point's geodetic coordinates. This makes the calculation of the topographic coordinate system easier to accomplish.

The first step of this transform is to determine to geodetic coordinates of an offset point (O_G) to the reference point (R_G). This point is used in determining the orientation of the topographic coordinate system. The offset point is a point at some offset $\Delta Elev$ above the reference point.

$$O_G = R_G + (0,0,\Delta Elev)$$

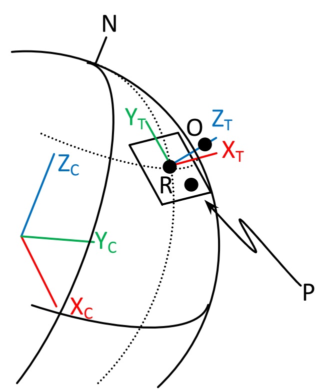


Figure 8: Topographic Coordinate System

The offset and reference point are then converted to geocentric coordinates using the method detailed in section 6.1.5.

$$O_C = \text{Geodetic\_to\_Geocentric\_Transform}(O_G)$$
$$R_C = \text{Geodetic\_to\_Geocentric\_Transform}(R_G)$$

These two points now define the normal vector to the ellipsoid so the topographic z-axis direction can be found by the difference of the reference point and the offset point.

$$\hat{Z}_T = \frac{O_C - R_C}{|O_C - R_C|}$$

The x and y-axes are solved for and the geocentric-to-topographic transformation matrix is built using the methods defined in section 6.1.7.

$$Xform = \begin{bmatrix} X_{T,x} & X_{T,y} & X_{T,z} & (-X_{T,x} \cdot R_{C,x} - X_{T,y} \cdot R_{C,y} - X_{T,z} \cdot R_{C,z}) \\ Y_{T,x} & Y_{T,y} & Y_{T,z} & (-Y_{T,x} \cdot R_{C,x} - Y_{T,y} \cdot R_{C,y} - Y_{T,z} \cdot R_{C,z}) \\ Z_{T,x} & Z_{T,y} & Z_{T,z} & (-Z_{T,x} \cdot R_{C,x} - Z_{T,y} \cdot R_{C,y} - Z_{T,z} \cdot R_{C,z}) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Any geodetic points can be converted to the topographic system by first converting them to geocentric coordinates using the method detailed in 6.1.5 and then transforming them to topographic coordinates using the transform defined above.

$$P_C = \text{Geodetic\_to\_Geocentric\_Transform}(P_G)$$

$$\begin{bmatrix} x_T \\ y_T \\ z_T \\ 1 \end{bmatrix} = \begin{bmatrix} X_{T,x} & X_{T,y} & X_{T,z} & (-X_{T,x} \cdot R_{C,x} - X_{T,y} \cdot R_{C,y} - X_{T,z} \cdot R_{C,z}) \\ Y_{T,x} & Y_{T,y} & Y_{T,z} & (-Y_{T,x} \cdot R_{C,x} - Y_{T,y} \cdot R_{C,y} - Y_{T,z} \cdot R_{C,z}) \\ Z_{T,x} & Z_{T,y} & Z_{T,z} & (-Z_{T,x} \cdot R_{C,x} - Z_{T,y} \cdot R_{C,y} - Z_{T,z} \cdot R_{C,z}) \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_C \\ y_C \\ z_C \\ 1 \end{bmatrix}$$

6.1.10 Topographic-to-Geodetic Transform

Description: Converts a topographic point in ENU to a geodetic point in WGS-84.

Following the logic laid out in section 6.1.8, the topographic-to-geocentric transform is the inverse of the geocentric-to-topographic transform. Given a topographic point $P_T \equiv (x_T, y_T, z_T)$, the geocentric equivalent $P_C \equiv (x_C, y_C, z_C)$ is calculated by:

$$\begin{bmatrix} x_C \\ y_C \\ z_C \\ 1 \end{bmatrix} = \begin{bmatrix} X_{T,x} & Y_{T,x} & Z_{T,x} & R_x \\ X_{T,y} & Y_{T,y} & Z_{T,y} & R_y \\ X_{T,z} & Y_{T,z} & Z_{T,z} & R_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_T \\ y_T \\ z_T \\ 1 \end{bmatrix}$$

The geocentric-to-geodetic transform is the same as the one detailed in section 6.1.6. The geocentric point is converted to a geodetic point P_G.

$$P_G = \text{Geocentric\_to\_Geodetic\_Transform}(P_C)$$

6.1.11 Geodetic Coordinate Conversions

6.1.11.1 Decimal Degrees to Degrees, Minutes, Seconds

Description: Converts a geodetic point in decimal degrees to a geodetic point in Degrees, Minutes, Seconds (DMS) representation.

A point in decimal degrees represented by:

$$(C_{Lat}\ ;\ C_{Lon}\ ;\ Elev)$$

$C \equiv$ Decimal Degrees
$Elev \equiv$ Elevation

Can be converted to a point in DMS coordinates:

$$(H_{Lat}, D_{Lat}, M_{Lat}, S_{Lat}\ ; H_{Lon}, D_{Lon}, M_{Lon}, S_{Lon}\ ; Elev)$$

$H \equiv$ DMS Hemisphere
$D \equiv$ DMS Degrees
$M \equiv$ DMS Minutes
$S \equiv$ DMS Seconds
$Elev \equiv$ Elevation

By first determining the hemispheres for latitude and longitude. The signs of C_{Lat} and C_{Lon} tell which hemispheres the coordinate lies in. The table below shows the hemisphere mapping.

Latitude	Longitude
$C_{Lat} \geq 0 \ \rightarrow \ H_{Lat} = N$	$C_{Lon} \geq 0 \ \rightarrow \ H_{Lon} = E$
$C_{Lat} < 0 \ \rightarrow \ H_{Lat} = S$	$C_{Lon} < 0 \ \rightarrow \ H_{Lon} = W$

Once the direction has been determined, the sign is stripped from the coordinate to simplify the remaining calculations.

$$C_{Lat} = |C_{Lat}|$$
$$C_{Lon} = |C_{Lon}|$$

Degrees, minutes, and seconds can then be calculated.

$$D_{Lat} = \text{floor}(C_{Lat})$$
$$D_{Lon} = \text{floor}(C_{Lon})$$

$$M_{Lat} = \text{floor}\big((C_{Lat} - D_{Lat}) \cdot 60\big)$$
$$M_{Lon} = \text{floor}\big((C_{Lon} - D_{Lon}) \cdot 60\big)$$

$$S_{Lat} = \text{floor}\left(\big((C_{Lat} - D_{Lat}) \cdot 60\big) - M_{Lat}\right) \cdot 60$$
$$S_{Lon} = \text{floor}\left(\big((C_{Lon} - D_{Lon}) \cdot 60\big) - M_{Lon}\right) \cdot 60$$

The valid ranges for the inputs C_{Lat} and C_{Lon} are:

$$-90^\circ \leq C_{Lat} \leq 90^\circ \qquad\qquad -180^\circ \leq C_{Lon} \leq 180^\circ$$

6.1.11.2 Degrees, Minutes, Seconds to Decimal Degrees

Description: Converts a geodetic point in Degrees, Minutes, Seconds (DMS) to a geodetic point in decimal degrees representation.

A point in DMS coordinates represented by:

$$(H_{Lat}, D_{Lat}, M_{Lat}, S_{Lat}\ ; H_{Lon}, D_{Lon}, M_{Lon}, S_{Lon}\ ; Elev)$$

$H \equiv$ DMS Hemisphere

$D \equiv$ DMS Degrees
$M \equiv$ DMS Minutes
$S \equiv$ DMS Seconds
$Elev \equiv$ Elevation

Can be converted to a point in decimal coordinates:

$$(C_{Lat}\ ;\ C_{Lon}\ ;\ Elev)$$

$C \equiv$ Decimal Degrees
$Elev \equiv$ Elevation

By first combining the values of degrees, minutes, and seconds into a decimal coordinate.

$$C_{Lat} = D_{Lat} + \frac{M_{Lat}}{60} + \frac{S_{Lat}}{3600}$$
$$C_{Lon} = D_{Lon} + \frac{M_{Lon}}{60} + \frac{S_{Lon}}{3600}$$

The sign of C_{Lat} and C_{Lon} are set from the hemisphere variables in the DMS representation. The table below shows the hemisphere mapping.

Latitude	Longitude
$H_{Lat} = N \rightarrow C_{Lat} = (+)$	$H_{Lon} = E \rightarrow C_{Lon} = (+)$
$H_{Lat} = S \rightarrow C_{Lat} = (-)$	$H_{Lon} = W \rightarrow C_{Lon} = (-)$

The valid ranges for the inputs are:

$$H_{Lat} = \{N, S\}$$
$$0 \le D_{Lat} \le 90$$
$$0 \le M_{Lat} \le 59$$
$$0 \le S_{Lat} \le 59.\bar{9}$$

$$H_{Lon} = \{E, W\}$$
$$0 \le D_{Lon} \le 180$$
$$0 \le M_{Lon} \le 59$$
$$0 \le S_{Lon} \le 59.\bar{9}$$

If $D_{Lat} = 90$ or $D_{Lon} = 180$ then the minutes and seconds must be zero.

6.2 Mensuration

In the field of photogrammetry, mensuration is the measurement of geometric quantities from imagery. In particular, measurement of linear distance and other mathematical properties based on length (i.e. area, circumference, etc.)

6.2.1 Rapid Positioning Capability

Rapid positioning capability (RPC), also known as the rational polynomial model and rational polynomial coefficients, is an approximation to the rigorous sensor model for determining the mapping of geodetic ground points in WGS-84 to the imaging system's pixel coordinates. The rapid positioning capability uses a pair of polynomial ratios to approximate the complex geometric relationship between the camera and the surface of the Earth.

6.2.1.1 RPC Basics

The RPCs map the normalized geodetic coordinates to normalized image coordinates. This means that the quantities being converted using the rational functions are always between 0 and 1. This serves as a good indicator to whether a point lies inside the valid range for the RPCs or if it's being extrapolated. The conversions to and from the normalized are listed below.

Normalized	Expanded
$y = \frac{(Lat - \text{Lat\_Offset})}{\text{Lat\_Scale}}$	$Lat = y \cdot \text{Lat\_Scale} + \text{Lat\_Offset}$
$x = \frac{(Lon - \text{Lon\_Offset})}{\text{Lon\_Scale}}$	$Lon = x \cdot \text{Lon\_Scale} + \text{Lon\_Offset}$
$z = \frac{(Elev - \text{Height\_Offset})}{\text{Height\_Scale}}$	$Elev = z \cdot \text{Height\_Scale} + \text{Height\_Offset}$
$l = \frac{Row - \text{Line\_Offset}}{\text{Line\_Scale}}$	$Row = l \cdot \text{Line\_Scale} + \text{Line\_Offset}$
$s = \frac{Col - \text{Samp\_Offset}}{\text{Samp\_Scale}}$	$Col = s \cdot \text{Samp\_Scale} + \text{Samp\_Offset}$

The offsets and scales referenced in the equations are quantities delivered with the RPCs. These values describe the bounding region that the RPCs are valid over. The figure below shows the viewing volume of the RPCs.

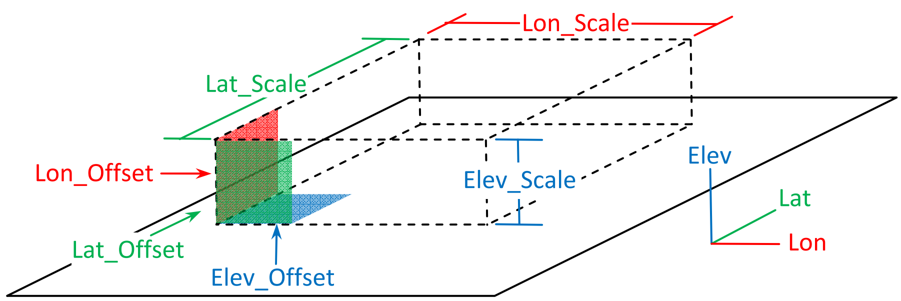


Figure 9: RPC Interpolation Volume

The offset terms specify the starting edge of the viewing volume and the scales determine the height, width, and depth. The same relationship is true for the image coordinates as well.

Another thing to note is that the latitude maps to the y-coordinate and the longitude maps to the x-coordinate. This can be a potential problem source when using geodetic coordinates. Typical geodetic coordinate representation is displayed as latitude, longitude, and elevation, but to define a proper right-handed coordinate system, coordinates must be used as longitude, latitude, elevation. Speaking from experience, this can be a significant issue for a mensuration engine when people have different viewpoints on how coordinates should be represented.

The rational polynomial equations convert geodetic coordinates to image coordinates using a pair of polynomial ratios. The equations are 3rd order polynomials in x, y, and z with 20 terms. The normalized line and sample location for a given ground coordinate are found using the equations:

$$l = \frac{C_{L,Num} \circ \rho}{C_{L,Den} \circ \rho} \qquad s = \frac{C_{S,Num} \circ \rho}{C_{S,Den} \circ \rho}$$

Where $C_{L,Num}$, $C_{L,Den}$, $C_{S,Num}$, and $C_{S,Den}$ represent the coefficient arrays for the line numerator, line denominator, sample numerator, and sample denominator, respectively. ρ represents the normalized geodetic terms associated with the coefficients. For RPC00B, ρ is in the form of:

$$\rho = [\; 1 \;\; x \;\; y \;\; z \;\; xy \;\; xz \;\; yz \;\; x^2 \;\; y^2 \;\; z^2 \;\; \ldots \;\; xyz \;\; x^3 \;\; xy^2 \;\; xz^2 \;\; x^2y \;\; y^3 \;\; yz^2 \;\; x^2z \;\; y^2z \;\; z^3 \;]$$

The full numerators and denominators for each equation are found by taking the dot product of the coefficient vector and the normalized geodetic term vector.

6.2.1.2 RPC00B Specification

All of the information required to use the rapid positioning capability is included in the RPC00B in the STDi-0002 spec. This information includes:

ERR_BIAS	Bias in RPC error. Assumes a 68% non time-varying error for correlated images.
ERR_RAND	Random component in RPC error. Assumes a 68% non time-varying error for correlated images.
LINE_OFF	Line offset
SAMP_OFF	Sample offset
LAT_OFF	Geodetic latitude offset
LONG_OFF	Geodetic longitude offset
HEIGHT_OFF	Geodetic height offset (measured in height above ellipsoid)
LINE_SCALE	Line scale
SAMP_SCALE	Sample scale
LAT_SCALE	Geodetic latitude scale
LONG_SCALE	Geodetic longitude scale
HEIGHT_SCALE	Geodetic height scale (measured in height above ellipsoid)
LINE_NUM_COEFF_1 … LINE_NUM_COEFF_20	Line numerator coefficients. The twenty coefficients for the polynomial in the numerator of the l equation.
LINE_DEN_COEFF_1 … LINE_DEN_COEFF_20	Line denominator coefficients. The twenty coefficients for the polynomial in the denominator of the l equation.
SAMP_NUM_COEFF_1 … SAMP_NUM_COEFF_20	Sample numerator coefficients. The twenty coefficients for the polynomial in the numerator of the s equation.
SAMP_DEN_COEFF_1 … SAMP_DEN_COEFF_20	Sample denominator coefficients. The twenty coefficients for the polynomial in the denominator of the s equation.

RPC00B specification pulled from STDI-0002, Reference item 4.

6.2.2 Geodetic-to-Image Transform

<u>Description</u>: Converts a geodetic point in WGS-84 to an image coordinate using the rational polynomial coefficients.

Given a geodetic point $P_G \equiv (Lat, Lon, Elev)$ the image coordinate $P_I \equiv (Col, Row)$ can be found using the rational polynomial function. The offsets, scales, and coefficients are loaded from the input image. Using the geodetic offsets and scales the unitized geodetic point can be found.

$$x = \frac{(Lon\ - \mathrm{Lon\_Offset})}{\mathrm{Lon\_Scale}}$$
$$y = \frac{(Lat - \mathrm{Lat\_Offset})}{\mathrm{Lat\_Scale}}$$
$$z = \frac{(Elev\ - \mathrm{Height\_Offset})}{\mathrm{Height\_Scale}}$$

The normalized geodetic coordinates are plugged into the normalized geodetic term array (ρ):

$$\rho = [\ 1\quad x\quad y\quad z\quad xy\quad xz\quad yz\quad x^2\quad y^2\quad z^2\ \ldots\quad xyz\quad x^3\quad xy^2\quad xz^2\quad x^2y\quad y^3\quad yz^2\quad x^2z\quad y^2z\quad z^3\ ]$$

The dot product of the coefficient arrays and the geodetic term arrays are taken to create the rational functions

$$l = \frac{C_{L,Num} \circ \rho}{C_{L,Den} \circ \rho} \qquad s = \frac{C_{S,Num} \circ \rho}{C_{S,Den} \circ \rho}$$

Where l is the normalized line and s is the normalized sample coordinate. These values are expanded to the full image coordinates Row and Col.

$$Row = l \cdot \text{Line\_Scale} + \text{Line\_Offset} \qquad Col = s \cdot \text{Samp\_Scale} + \text{Samp\_Offset}$$

Row and Col represent floating point pixel coordinates in image space.

6.2.3 Single Image, Image-to- Geodetic Transform

The image to geodetic function iteratively converges on a geodetic location using an initial estimation of ground position and an iterative technique to converge on the proper ground position at a constant elevation.

The first step of this method is to determine the relationship between image lines and samples and ground latitude and longitude. This can be achieved by taking the partial derivatives of the normalized mensuration equations. The normalized projection equations are shown below:

$$l = \frac{L}{M} = \frac{C_{L,Num} \circ \rho}{C_{L,Den} \circ \rho} \qquad s = \frac{N}{P} = \frac{C_{S,Num} \circ \rho}{C_{S,Den} \circ \rho}$$

Where l and s represent the lines and samples of the image, the C variables represent the coefficient arrays for each numerator and denominator, and the quantity ρ is the normalized geodetic term array. The partial derivatives of each numerator and denominator must be calculated. Using the numerator of the lines rational function, the partial derivatives are formulated by:

$$\frac{\partial L}{\partial x} = C_{L,Num} \circ \frac{\partial \rho}{\partial x} \qquad \frac{\partial L}{\partial y} = C_{L,Num} \circ \frac{\partial \rho}{\partial y}$$

Since $C_{L,Num}$ contains only coefficients, it can be pulled outside of the derivative leaving only the partials of the normalized geodetic array. The same procedure can be used for each numerator and denominator, so the calculus reduces to only the partial derivatives of ρ with respect to x and y.

$$\frac{\partial \rho}{\partial x} = [\ 0\quad 1\quad 0\quad 0\quad y\quad z\quad 0\quad 2x\quad 0\quad 0\ \ldots\quad yz\quad 3x^2\quad y^2\quad z^2\quad 2xy\quad 0\quad 0\quad 2xz\quad 0\quad 0\ ]$$

$$\frac{\partial \rho}{\partial y} = [\ 0\quad 0\quad 1\quad 0\quad x\quad 0\quad z\quad 0\quad 2y\quad 0\ \ldots\quad xz\quad 0\quad 2xy\quad 0\quad x^2\quad 3y^2\quad z^2\quad 0\quad 2yz\quad 0\ ]$$

The partial derivates of l and s with respect to longitude and latitude (x and y) are then calculated by applying the quotient rule in calculus.

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x)f(x)' - f(x)g(x)'}{g(x)^2}$$

The partial derivates of line are:

$$\frac{\partial l}{\partial x} = \frac{M \cdot \frac{\partial L}{\partial x} - L \cdot \frac{\partial M}{\partial x}}{M^2} \qquad \frac{\partial l}{\partial y} = \frac{M \cdot \frac{\partial L}{\partial y} - L \cdot \frac{\partial M}{\partial y}}{M^2}$$

And the partials of sample are:

$$\frac{\partial s}{\partial x} = \frac{P \cdot \frac{\partial N}{\partial x} - N \cdot \frac{\partial P}{\partial x}}{P^2} \qquad \frac{\partial s}{\partial y} = \frac{P \cdot \frac{\partial N}{\partial y} - N \cdot \frac{\partial P}{\partial y}}{P^2}$$

These quantities show how line and sample values change with respect to latitude and longitude. For the image-to-ground projection, though, the inverse is needed. These values can be calculated by simply inverting each of the quantities.

The iterative method for calculating ground location starts with an initial guess and calculates the corresponding image location.

$$(l_0, s_0) = \mathrm{Ground\_To\_Image}(\mathrm{X}_0, \mathrm{Y}_0, \mathrm{Z}_0)$$

The error in image space between the desired pixel location and the initial guess is calculated.

$$\Delta l_i = l_{act} - l_i$$
$$\Delta s_i = s_{act} - s_i$$

This delta in image space can be translated to an approximate delta in geodetic space by calculating the change in latitude and longitude with respect to line and sample.

$$x_{i+1} = \frac{\partial x}{\partial l} \cdot \Delta l_i + \frac{\partial x}{\partial s} \cdot \Delta s_i$$
$$y_{i+1} = \frac{\partial y}{\partial l} \cdot \Delta l_i + \frac{\partial y}{\partial s} \cdot \Delta s_i$$

This can be converted to matrix form, where the delta in image space is multiplied by the Jacobian matrix.

$$\begin{bmatrix} x_{i+1} \\ y_{i+1} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial l} & \frac{\partial x}{\partial s} \\ \frac{\partial y}{\partial l} & \frac{\partial y}{\partial s} \end{bmatrix} \cdot \begin{bmatrix} \Delta l_i \\ \Delta s_i \end{bmatrix}$$

This procedure is iterated until Δl_i and Δs_i fall beneath a set threshold. This threshold can vary according to the desired accuracy of the projection, but is typically set to 10^{-6} or less.

6.2.4 Multiple Image, Image-to- Geodetic Transform

[TBD]

6.3 Satellite Image Geometry

[TBD]

6.3.1 Line-of-Sight Vector (LOS)

[TBD]

6.3.2 Azimuth and Elevation

[TBD]

6.3.3 Ground Sample Distance

[TBD]

6.4 Registration

[TBD]

6.4.1 Absolute Tie Point Registration (Image-to-GCP)

[TBD]

6.4.2 Relative Tie Point Registration (Image-to-Image Tie Point)

[TBD]

6.4.3 Relative Image Co-registration (Image-to-Image Using Image Contents)

[TBD]

6.5 Rectification

[TBD]

6.5.1 Georectification

[TBD]

6.5.2 Epipolar Rectification

Epipolar rectification is the process of aligning a pair of image such that the parallax due to elevation runs horizontally in the image.

Figure 10: Epipolar Stereo Pair *(Courtesy of GeoEye, Inc.)*

6.5.2.1 Determine Reference Point and Build Topographic Coordinate System

The first step in creating an epipolar stereo pair is determining the reference point of the system. The reference point will be used to create the topographic coordinate system and will be one of the defining points of the epipolar plane. Figure 11 shows a stereo image topographic coordinate system with the reference point depicted by R and the satellite locations during acquisition shown by S_1 and S_2.

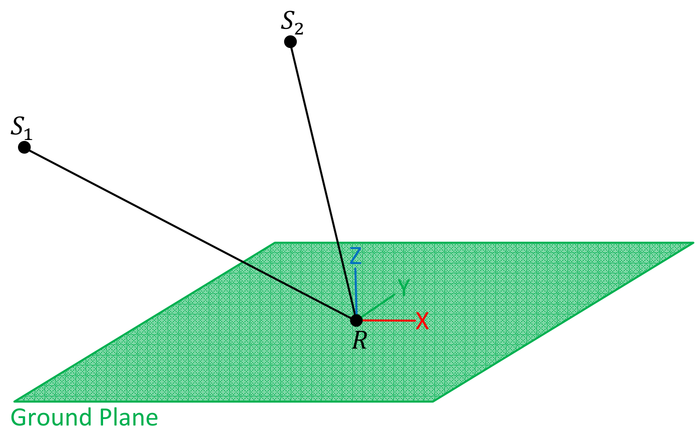


Figure 11: Topographic Coordinate System

R represents a geodetic point anywhere on the Earth. The reference point is typically chosen such that after rectification, R will be located in the center of the new images. Since the epipolar geometry is slightly different at each ground location, placing R in the center will minimize the visible effects of the changing geometry in the output images.

The ground plane is defined as a plane running through the reference point, normal to the topographic system's z-axis. Since the ground plane will always run through the topographic origin, the planar equation is simply:

$$z = 0$$

or in homogeneous coordinates.

$$[0 \quad 0 \quad 1 \quad ; \quad 0]$$

The location of the topographic coordinate system location is needed to create the spatial transforms in the following sections.

6.5.2.2 Calculate Offset Points and Line of Sight Vectors

Now that the topographic coordinate system is defined, the next requirement is to calculate the epipolar plane. The definition of a plane requires three points. The first of these points is R since by definition, the epipolar plane has to pass through R. If they were known S_1 and S_2 could be used to complete the planar definition, however, this requires knowing the satellite ephemeris at the time of collection. This data is not typically available, so the process defaults to data that is normally delivered with satellite imagery, the rational polynomial coefficients (RPCs).

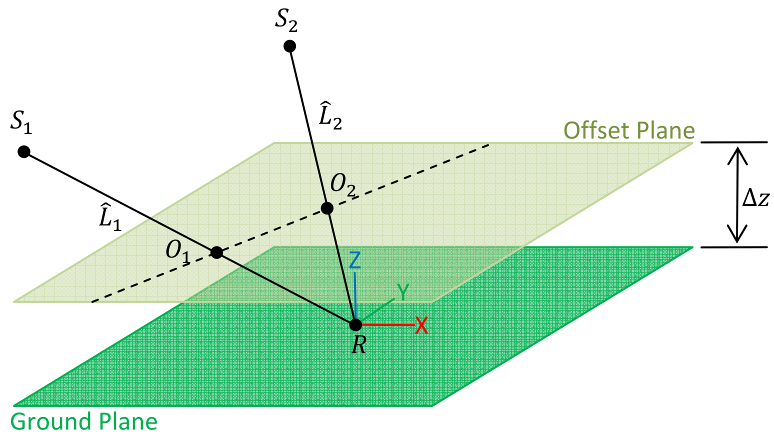


Figure 12: Offset Points

Using the RPCs a second set of points O_1 and O_2, known as the offset points, can be calculated, see Figure 12. The offset points are determined by first finding the reference point in the source images. Using the mensuration equations of each image, the pixel coordinates of the reference point R can be found.

$$I_1 = \text{Image1.GroundToImage}(R)$$
$$I_2 = \text{Image2.GroundToImage}(R)$$

Then using the ImageToGround projections at an elevation offset from the ground, the offset points can be calculated.

$$O_1 = \text{Image1.GroundToImage}(I_1, R.Elev + \Delta z)$$
$$O_2 = \text{Image2.GroundToImage}(I_2, R.Elev + \Delta z)$$

All three points must then be converted to the local topographic system. Using the method laid out in Section 6.1.9 on page 11, the geodetic reference point and offset points can be converted to topographic coordinates.

$$O_{1,\text{Topo}} = \text{GeodeticToTopographic}(O_{1,\text{Geo}})$$
$$O_{2,\text{Topo}} = \text{GeodeticToTopographic}(O_{2,\text{Geo}})$$

Since the coordinate system origin is defined at the reference point, R will always be $(0,0,0)$. Using the topographic points the LOS vectors can be calculated.

$$\hat{L}_1 = \frac{R_{\text{Topo}} - O_{1,\text{Topo}}}{\left|R_{\text{Topo}} - O_{1,\text{Topo}}\right|}$$
$$\hat{L}_2 = \frac{R_{\text{Topo}} - O_{2,\text{Topo}}}{\left|R_{\text{Topo}} - O_{2,\text{Topo}}\right|}$$

$\hat{L}_1$ and $\hat{L}_2$ are normalized by dividing each vector by its magnitude.

6.5.2.3 Calculate Epipolar Plane

The epipolar plane can now be calculated using R, O_1, and O_2, see Figure 13. <u>Note</u>*: All subsequent calculations will be in topographic coordinates unless otherwise stated.*

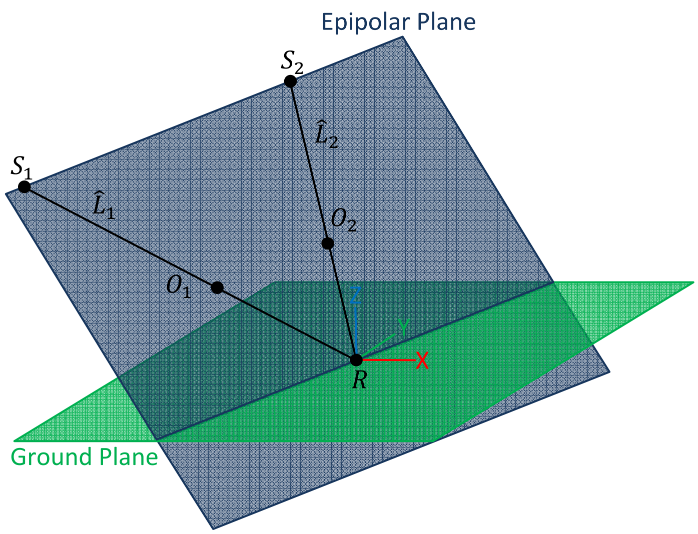


Figure 13: Epipolar Plane

Since the LOS vectors are contained in the epipolar plane, the cross product of the two vectors will give the normal direction of the plane.

$$\hat{N} = \hat{L}_1 \times \hat{L}_2$$

Using the typical equation for a plane:

$$Ax + By + Cz + D = 0$$

The plane normal $\hat{N}$ corresponds to the A, B, and C coefficients. To calculate the final planar coefficient, the values of R are plugged into the planar equation and solved for D.

$$D = -\hat{N}_x \cdot R_x - \hat{N}_y \cdot R_y - \hat{N}_z \cdot R_z$$

6.5.2.4 Calculate Epipolar Line

The epipolar line is calculated by intersecting the ground plane and the epipolar planes in topographic space. The epipolar line represents the horizontal viewing axis of the two cameras when used as a single binocular system. Images that are displayed such that the epipolar line runs horizontally across the screen will have elevation changes appear to move horizontally in the image. Typically, the epipolar line is projected back to image space, but for these calculations, it's easier to handle all of the calculations in topographic space and only convert back to image space when the interpolation is being done. The epipolar line is shown in Figure 14.

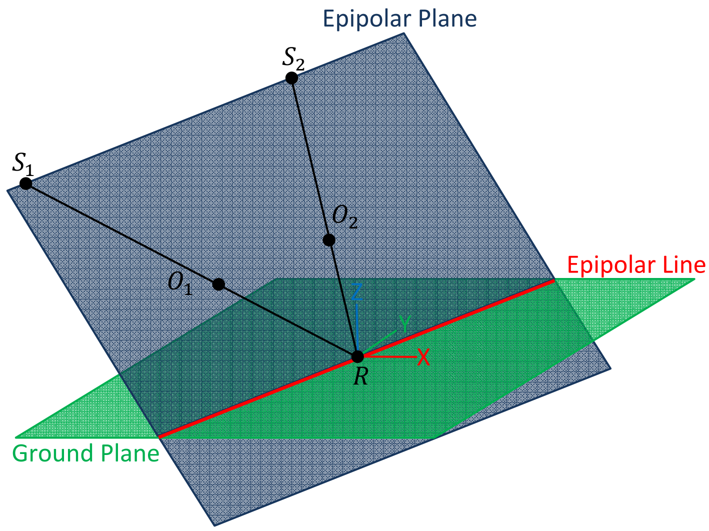


Figure 14: Epipolar Line

Note: The following calculations rely on projective geometry concepts that can be found in the Geometry chapter.

Calculating the intersection of the epipolar and ground planes is performed quite easily using projective geometry. Using the projective geometry definition of a line from two planes, the epipolar line can be found in Plücker coordinates.

$$\{\hat{N}_G \times \hat{N}_E \quad ; \quad D_E\hat{N}_G - D_G\hat{N}_E\}$$

Where $\hat{N}_G \times \hat{N}_E$ gives the direction of the line, and $D_E\hat{N}_G - D_G\hat{N}_E$ gives the moment. Broken down into Plücker component form, the equations are:

$$\begin{aligned} D_x &= B_G C_E - B_E C_G \\ D_y &= C_G A_E - C_E A_G \\ D_z &= A_G B_E - A_E B_G \\ M_x &= D_G A_E - D_E A_G \\ M_y &= D_G B_E - D_E B_G \\ M_z &= D_G C_E - D_E C_G \end{aligned}$$

Where $< D_x, D_y, D_z >$ is the direction unit vector of the line and $< M_x, M_y, M_z >$ represents the moment of inertia of the line.

6.5.2.5 Build Pixel Grid

Now that the epipolar line is defined, the pixel grid can be defined. The pixel grid must align with the epipolar line in topographic space and should extend outward from the reference point. The spacing between each pixel, or ground sample distance (GSD), is user defined. In theory, the GSD can be set to any value, but in practice should be set to a value close to the GSD of the original images.

Using a sharper GSD for the output than is available in the input images will return an image that has more significant interpolation blurring. This is from oversampling the image signal and filling the gaps with interpolated data. Using a coarser GSD than the input can cause aliasing when interpolating, which can make an image look grainy and pixilated.

Note: For more information on interpolation effects, see the Image Processing chapter.

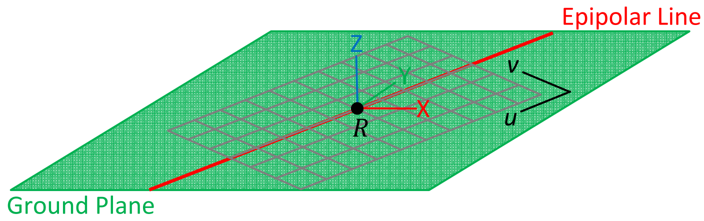


Figure 15: Pixel Grid

The pixel grid is defined in 3D spatial coordinates on the topographic ground plane, see Figure 15. The image coordinate system is measured in pixel coordinates (u, v) starting from the top left corner of the image. The extents of the pixel grid should be $(n-1) \cdot GSD$ where n is the number of pixels and GSD is the desired ground sample distance of the new image. The image footprint is shown in Figure 16.

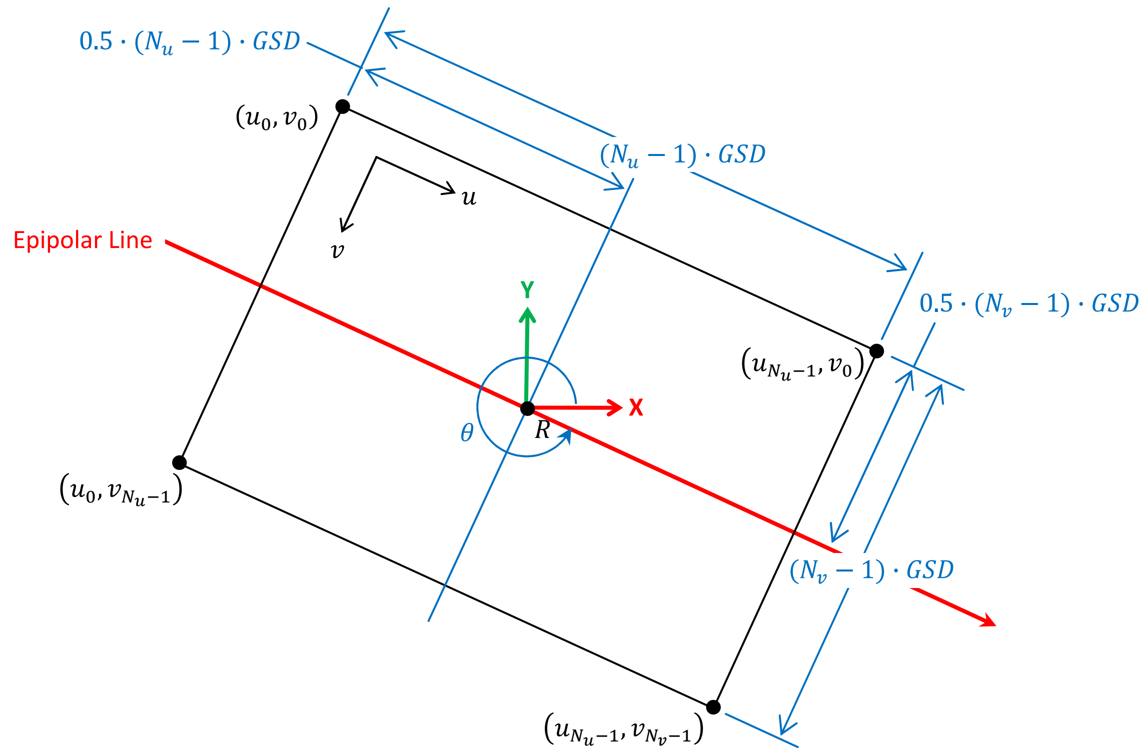


Figure 16: Image Footprint and Dimensions

To determine the pixel locations, the u and v vectors must be calculated. u is simply the x,y components of the epipolar line's direction vector.

$$u = (D_x, D_y)$$

<u>Note</u>: The epipolar line should have a z-component of zero, but because of numerical roundoff, this value may be non-zero. If so, u may have to be re-normalized.

The v vector can be found by taking the normal of u using the equation:

$$V'(x, y) = V(y, -x)$$

Since this system has the z-axis pointing downward instead of the typical upward convention, the normal is the negative.

$$v(x, y) = u(-y, x)$$

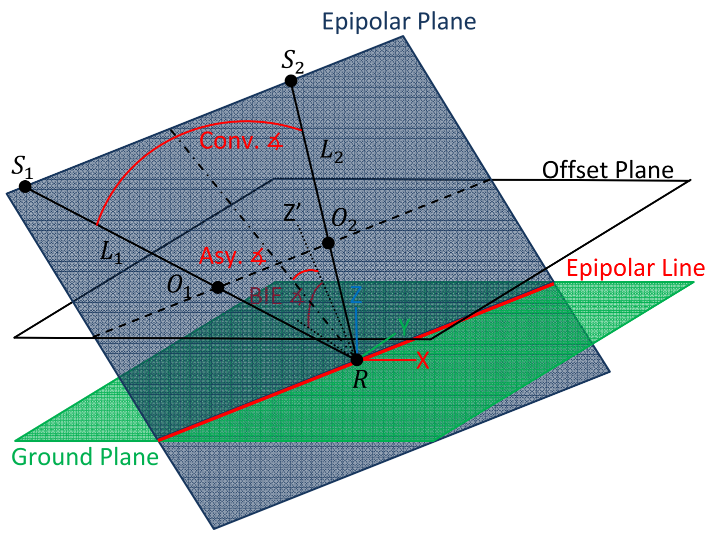
@@@@

6.5.2.6 Interpolate New Images

@@@@

6.5.2.7 Determine Left Image from Right

@@@@



6.5.2.8 Calculate Stereo Angles

There are three main angles of interest that define a stereo pair: the convergence angle, the asymmetry angle, and the bisector elevation angle. These three angles are shown in Figure 17 below indicated by Conv. ∡, Asy. ∡, and BIE ∡, respectively.

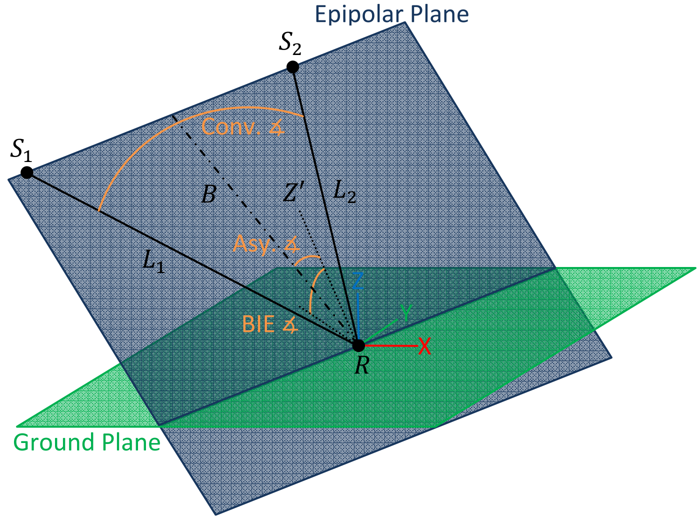


Figure 17: Stereo Angles

Convergence Angle

The most important of the three angels is the convergence. This angle represents the apparent point of view change between the two images. Typical human binocular vision involves viewing objects at convergence angles from nearly 0° (for far away objects) to 30° (for close up objects). As the distance between the object being viewed and our eyes decrease, the convergence angle increases. Viewing objects with a convergence angle of greater than 30° begins to cause strain on the eyes.

The same rules apply for stereo imagery. For visually appealing stereo imagery, a convergence angle of around 15° is comfortable to look at. Conversely, if elevations are being extracted from a stereo pair, a small convergence angle will cause larger elevation errors. For these cases, a convergence angle of around 30° is desirable, see Figure 18.

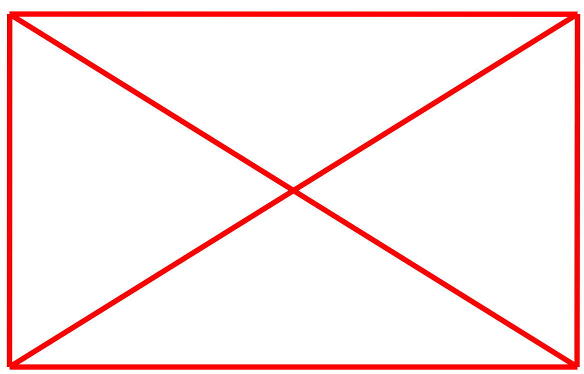


Figure 18: Convergence Angle Comparison (15° on Top, 30° on Bottom)

The convergence angle can be calculated using the LOS vectors L_1 and L_2 calculated earlier. Using the formula for the dot product:

$$u \circ v = |u||v| \cos(\theta)$$

Since L_1 and L_2 are unit vectors the equation reduces to

$$L_1 \circ L_2 = \cos(Conv \measuredangle)$$

Solving for θ:

$$Conv \measuredangle = \cos^{-1}(L_1 \circ L_2)$$

Asymmetry Angle

The second most important stereo angle is the asymmetry angle. Asymmetry describes the apparent offset from the center view that a stereo pair has. For instance, a stereo pair with an asymmetry of 0° will have the parallax due to elevation appear equivalent in the left and right images. However, a stereo pair with an asymmetry of 15° will have one of the images appear to have greater parallax than the other, see Figure 19.

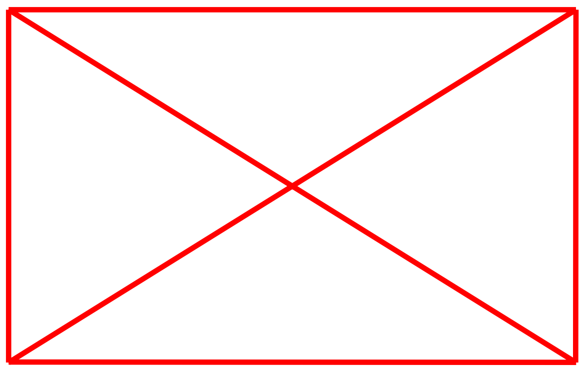


Figure 19: Asymmetry Angle Comparison (0° on Top, 15° on Bottom)

The asymmetry angle calculation depends on the vector Z' which is a projection of the topographic coordinate system's z-axis onto the epipolar plane. The direction of Z' can be found by taking the triple product of the plane's normal vector $(\hat{N})$ and the z-axis (Z).

$$Z' = \hat{N} \times Z \times \hat{N}$$

Next the bisector vector (B) must be found. This is simply the average of the two LOS vectors.

$$B = \frac{L_1 + L_2}{|L_1 + L_2|}$$

Since the bisector vector should be a unit vector, the sum is divided by its magnitude. Finally, the asymmetry angle is calculated by determining the angle between the bisector vector and the projected z-axis. This is done using the same dot product technique detailed in the last section.

$$Asy \measuredangle = \cos^{-1}(B \circ Z')$$

Bisector Elevation Angle (BIE)

The BIE angle is the final angle of interest in a stereo pair. This angle depicts the amount of parallax will appear in the vertical direction after alignment. The BIE angle is the angle between the ground plane and the epipolar plane.

A BIE angle of 90° means the epipolar plane is perpendicular to the ground plane and all of the parallax in the image will run horizontally across the image. A BIE angle of 0° would mean the satellites were scanning tangent to the surface of the Earth. A typical BIE angle is going to range from 45° to 90°. The effects of the BIE angle are shown in Figure 20.

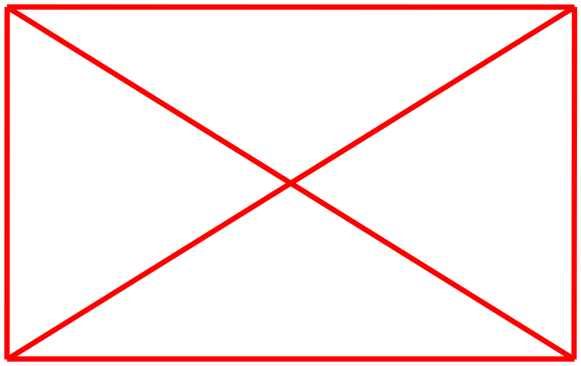


Figure 20: BIE Angle Comparison (90° on Top, 45° on Bottom)

The BIE angle is calculated simply as the angle between the ground and epipolar planes. This can be calculated using the dot product rule on the planes' normal vectors.

$$BIE \measuredangle = \cos^{-1}\left(\hat{N}_G \circ \hat{N}_E\right)$$

6.6 Stereo Disparity

Stereo disparity is the image-to-image mapping of points in one image to points in a second image. Most applications of disparity mapping require the images to be epipolar rectified and well registered in the y-direction, see Figure 21.

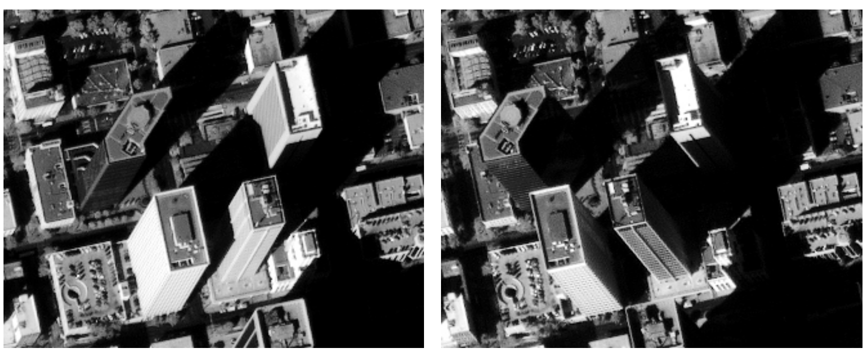


Figure 21: Epipolar Stereo Pair *(Courtesy of GeoEye, Inc.)*

In a well registered epipolar stereo pair, all of the motion between the two images should be in the horizontal direction. This allows the searching for correspondence between images to @@@

6.6.1 Correlation Techniques

Disparity mapping require

6.6.2 Range Estimation

It is useful to bound the range searched

6.6.2.1 Single Point Disparity Left-to-Right

[TBD]

6.6.2.2 Single Point Disparity Right-to-Left

[TBD]

6.6.2.3 Average and Standard Deviation Crop

[TBD]

6.6.3 Disparity Volume

[TBD]

6.6.3.1 Volume Left-to-Right Disparity

[TBD]

6.6.3.2 Volume Right-to-Left Disparity

[TBD]

6.6.3.3 Left-Right Comparison

[TBD]

6.6.3.4 Localized Average and Standard Deviation Crop

[TBD]

6.6.4 Disparity Growth

[TBD]

6.7 References

1 Weisstein, Eric W., *MathWorld—A Wolfram Web Resource*, "Mensuration Formula", http://mathworld.wolfram.com/

2 *Wikipedia, the free encyclopedia,* "Photogrammetry", "World Geodetic System", http://www.wikipedia.org/

3 *Univ. of Colorado – Department of Geography,* "Geodetic Datum Overview", *http://www.colorado.edu/geography/gcraft/notes/datum/datum.html*

4 National Geospatial-Intelligence Agency, *STDI-0002 – The Compendium of Controlled Extensions (CE) for the National Imagery Transmission Format (NITF)*

5 Dr. Kelso, T.S., *Center for Space Standards & Innovation – CelesTrack*, "Orbital Coordinate Systems, Part I", http://www.celestrak.com/columns/v02n01/

6 Department of Defense, *MIL-STD-2500B – National Imagery Transmission Format Version 2.1 for the National Imagery Transmission Format Standard*

7 @@@@ DISPARITY DOCUMENT @@@@